E-Learning Notes

On

Strength of Material

STRESS AND STRAIN

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Chapter – 1

Stress and Strain

Strength of material: The resistance offered by the material of body due to applied force (which test to define the body) is known as strength of material.

Load: An external force acting on a body.

Classification according the nature of application:

1) **Dead load or Study Load:** These are the load which are applied very gradually *i.e.* increase from zero to their maximum value. These load do not change their magnitude, direction and point of application. example roof, bridge.
2) **Live or Fluctuating Load:** A load is said to be life of fluctuating load, when it changes continuously of suddenly.
   a) **Variable Load**: A load is said to be variable when it changes continuously. Examples traffic crossing a bridge.
   b) **Suddenly Load**: A load is said to be applied or shock load, when it suddenly applied or removed.
   c) **Impact Load:** A load is said to be an impact load, when it is applied with some initial velocity. Example hammer blow.

Classification of load according to the effect produce on the member:

1) **Tensile Load**: The load which tend to pull the member in the direction of its application is called tensile load.
2) **Compressive Load:** The load which tends to push together the opposite end of the member is called compressive load.
3) **Share Load**: The load which tends to cause sliding of one face relative to the other, and consists of equal, parallel and opposite forces so as to form couple are called sharing road.
4) **Twisting load or Torsional Load**: The load produced by two couple applied at opposite ends of the member, tending to cause one end to rotate about its longitudinal axis relative to other end are called twisting or torsional load.
5) **Bending Load:** The load which tend to cause a certain degree of curvature or bending in the member are called bending loads.

Stress: The internal resistance per unit area of cross section is called stress. It is defined as the force per unit area.

$$\text{Stress } (\sigma) = \frac{\text{Force (P)}}{\text{Area (A)}}$$

Unit of stress: **- N/m², N/mm²**

$$\text{Stress } (\sigma) = \frac{\text{Force (P)}}{\text{Area (A)}} = \frac{\text{1 newton}}{(\text{1 metre})^2} = 1\ \text{N/m}^2$$

$$1\ \text{N/m}^2 = \frac{1\ N}{(1000mm)^2} = 10^{-6}\ \text{mm}^2$$

$$1\ \text{MPA} = 10^6 \times \text{N/m}^2$$

$$= 10^6 \times 10^{-6}\ \text{mm}^2 = 1\ \text{N/mm}^2$$

1 Pa(Pascal) = 1 N/m²

1 MPA = 1 N/mm²

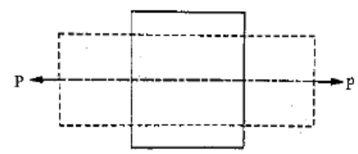
1 GPA = 10^3 × N/mm²

= 1 KN/mm²

Types of Stress: Stress can be classified into categories.

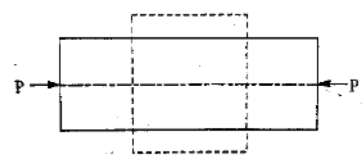
1. Direct Stress
2. Indirect Stress

1. **Direct stress**: when a force is applied perpendicular to the cross section of member the stress induced is known as direct stress and it is also called normal stress.
   a) **Tensile stress:** The stress produce in member when it is subjected to equal and opposite pulls is called tensile stress. The effect of tensile stress is increase the length of the body.



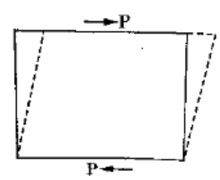
$$\sigma_t = \frac{P}{A}$$

   b) **Compressive stress**: The stress produce member when two equal and opposite force are applied on a body is called compressive stress. The effect of compressive stress is to decrease the length of the body.



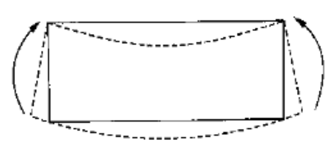
$$\sigma_C = \frac{P}{A}$$

   c) **Shear stress:** If two equal and opposite force applied tangentially to the cross section of a body the stress induced is known as shear stress. Shear stress is also called tangential stress.

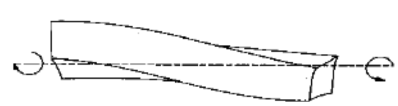


$$\tau = \frac{P}{A}$$

2. **Indirect Stress**: The stress which are developed internally to resist the deformation are called indirect stress.
   a) **Bending Stress:** The tensile and compressive stress induced due to bending on a beams in direction of their length are called bending stress. The bending stress is two type tensile bending stress and compressive bending stress. The axis of beam the at which stress is zero called neutral axis.



   b) **Torsional stress:** When two equal and opposite torques are applied ate the two ends of a shaft, the shaft is said to be torsion. The stress thus produced is called torsional stress. The torsional stress is equal to the product of the force applied tangentially to the ends of shaft and the radius of the shaft.



Stain: The ratio of change in dimension of the body to the original dimension is known is strain.

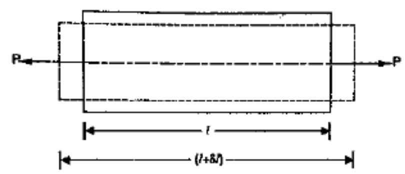
$$\text{Strain } (\epsilon) = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Types of strain

1. Tensile strain
2. Compressive strain
3. Shear strain
4. Volumetric strain

1. **Tensile strain:** The ratio of increase of dimension to the original dimension of body is known as tensile strain.

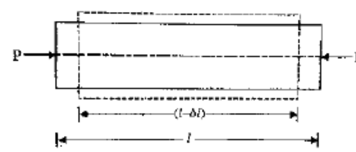
$$\text{Tensile Strain } (\epsilon_t) = \frac{\text{increase in length}}{\text{original length}} = \frac{(l + \delta l - l)}{l}$$



$$\epsilon_t = \frac{\delta l}{l}$$

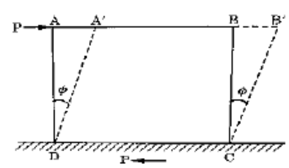
2. **Compressive strain:** The ratio of decrease in dimension to the original dimension off the body is known as compressive strain.

$$\text{Compressive Strain } (\epsilon_c) = \frac{\text{increase in length}}{\text{original length}} = \frac{(l - (l - \delta l)}{l}$$



$$\epsilon_c = \frac{\delta l}{l}$$

3. **Shear strain:** The strain produced by shear stress in known as shear strain.



$$\text{Shear Strain } (\epsilon_c) = \frac{CC_1}{BC} = \tan\phi = \phi$$

4. **Volumetric strain:** The ratio of change in volume to the original volume is known as volumetric strain.

$$\epsilon_V = \frac{\delta V}{V}$$

Elasticity: It is the property of material od a body by virtue of which it opposes any change being produced in its shape and size external force and its tend to regain its original shape and size after the removal of external force.

Elastic limit: The value of stress corresponding to this limiting force up to which the material is perfectly elastic is known as elastic limit.

Hook's Law: When a material loaded in elastic limit, the stress is directly proportional to the strain is known as hook's law.

$$(\sigma) \text{ Stress} \propto \text{Strain } (\epsilon)$$

Young modulus or Modulus of elasticity: It is ratio of tensile stress to the tensile strain

It is denoted by (E).

$$\text{Young Modulus (E)} = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\sigma_t}{\epsilon_t}$$

***Shear modulus or Modulus of rigidity*:** It is ratio of Shear stress to the Shear strain

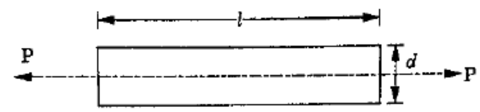
It is denoted by (G).

$$\text{Shear Modulus (G)} = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

Bulk Modulus: It is ratio to the normal stress to the volumetric strain. It is denoted by (K).

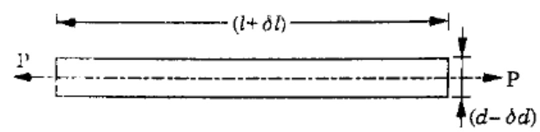
$$\text{Bulk Modulus (K)} = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{\sigma}{\epsilon_v}$$

Longitudinal Strain: The strain along the direction of the applied force is known as longitudinal strain or linear strain.



$$\text{Longitudinal strain} = \frac{\delta l}{l} \text{ (tensile)}$$

Lateral Strain: The strain at right angle to the direction of the applied force is known as lateral strain or transverse strain.



$$\text{Lateral strain} = \frac{\delta d}{d} \text{ (Compressive)}$$

Poisson's ratio: The ratio of lateral strain and longitudinal strain is known as Poisson's ratio. It is denoted by μ or $\frac{1}{m}$

$$\text{Poisson ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Proof Stress: Maximum stress required to cause a permanent extension to a definite percentage of gauge length is called proof stress. The proof load divided by original cross-section area gives 0.2% proof stress.

Ultimate stress: It is the ratio of maximum load to which a specimen is subjected in a tensile test and the original cross-sectional area of the specimen. It is also known as maximum stress.

$$\text{Ultimate stress} = \frac{\text{Maximum Load}}{\text{Original crosssection area}}$$

Working* stress*: It is the ratio of maximum stress to the factor of safety. The stress is also known as allowable stress and permissible stress.

$$\text{Working stress} = \frac{\text{Maximum stress}}{\text{Factor of safety}}$$

Factor of safety: (F.O.S.): It is the ratio of maximum stress to the working stress

$$\text{F.O.S.} = \frac{\text{Maximum Stress}}{\text{Working stress}}$$

Principle of superposition: When a number of loads are acting on a body, the resulting strain will be algebraic sum of the strains cause by the individual forces.

$$\delta l = \left(\frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} + \frac{P_3 l_3}{A_3 E_3}\right)$$

Thermal Stress: when there is a change in the temperature of a body, its length gets changed. If this change in length is prevented, then a stress is induced in the body. This stress is called thermal stress or temperature stress and corresponding strain is called temperature strain.

l = length of bar
t_1 =initial temperature of bar
t_2 = final temperature of bar
α = Coefficient of linear expansion

The extension in the bar due to rise in temperature will be

$$= \alpha (t_2 - t_1)\, l$$

If this elongation in the bar is prevented by some external force by fixed the bar ends, the temperature strain produced will be

$$\text{Temperature Strain} = \frac{\alpha (t2 - t1)\, l}{l} = \alpha (t_2 - t_1) \text{ (compressive)}$$

$$\text{Temperature stress} = \alpha (t_2 - t_1)\, E \text{ (compressive)}$$

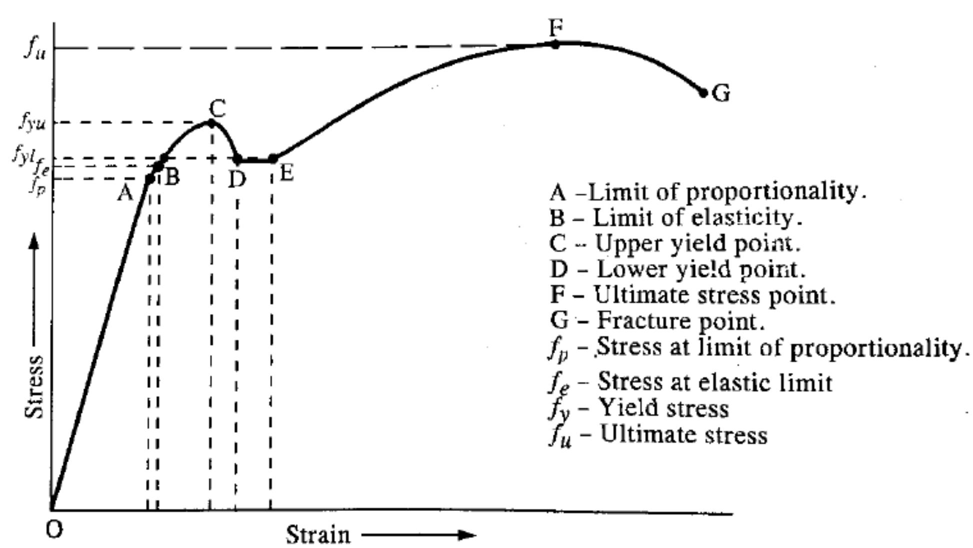
If the temperature od bar is lowered, the temperature stress and strain will be tensile.

Mechanical properties of material:

1) **Elasticity:** The ability of material which has been deformed in same way, to return to its original shape and size after the deformation force has been removed is called elasticity.
2) **Plasticity:** The ability of a material to flow to new shape under stress and retain its new shape is termed as plasticity. Plasticity is opposite to elasticity.
3) **Strength:** The ability off a material to withstand or support external loads without any structural damage is called strength.

4) **Stiffness:** The ability of material to resist deflection or elastic deformation under applied load is called stiffness or rigidity.
5) **Toughness:** The property of a material to offer resistance to fracture under shock loads such as hammer blow is called its toughness.
6) **Ductility:** The ability of material to be drawn its wire without rupture and without losing much strength is known as ductility.
7) **Malleability:** The ability of material to be deformed by being hammered, rolled or pressed without fracture through cold or hot working is termed as its malleability.
8) **Brittleness:** Lack of ductility is brittleness. When a body break easily when subjected to shocks it is said to be brittle.
9) **Hardness:** The property of material to resist scratching, abrasion, surface wear or indentation by harder bodies is called hardness.
10) **Fatigue:** The phenomenon that lead to fracture in metals and alloys under repeated fluctuating or alternating loads or stresses, too small to produced permanents deformation when applied statically, is called fatigue.
11) **Creep:** The property of material due to which its undergoes slow plastic deformation under prolonged loading, usually at high temperature, is called creep.

Stress and Strain diagram: In stress strain diagram, nominal stresses are calculated by dividing the loads by original cross-section and strain are calculated by dividing the extension or elongation by original length.



Stress–Strain curve for Ductile Materials

1. Limit of proportionality
2. Elastic limit
3. Yield point (upper yield point and lower yield point)
4. Ultimate stress point
5. Breaking point

1. **Limit of proportionality:** Point O to point A, the hook's law is obeyed. The limit up to which the stress is directly proportional to strain called limit of proportionality.
2. **Elastic limit:** On the removal of load, no permanent deformation is observed and material will regain its original shape and size. The point B is called elastic limit point.
3. **Yield Point:** Beyond the point B, the material goes to plastic stage until the upper yield point C is reached *i.e.* the removal of load does not allow to return the specimen to its original form. Point C is upper yield point. At this point, the cross-section area of the material starts decreasing and then the stress decreases to a lower value at the point D, called lower yield point.

   From point D, the specimen elongates by a considerable amount, without any increase in stress, up to the point E. The portion DE is called yielding of the material at constant stress.
4. **Ultimate stress point:** From point E onwards, the stress again starts increasing up to point F because the material picks up the ability to resist increasing stress., but the elongation not increasing at a much faster rate. The point is called ultimate stress point.
5. **Breaking point:** Beyond the point F, elongation will continue at gradually decreasing stresses due to local necking. It means extension of specimen continues even with a lesser and lesser load and ultimately the specimen breaks at the point G. Point G is known as breaking point or fracture point.

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STRAIN ENERGY
OR
RESILIENCE

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Chapter – 2

Resilience or Strain Energy

Strain energy: the work done straining the body within the elastic limit is known as strain energy.

Strain energy = Work done

Resilience: The strain energy stored by the body within the elastic limit when loaded externally is called resilience.

Proof resilience: The maximum strain energy which can be stored in a body up to the elastic limit is called proof resilience.

Modulus of resilience: Proof resilience per unit volume the body is known as modulus od resilience.

$$\text{Modulus of Resilience} = \frac{\text{Proof Resilience}}{\text{Volume of the body}}$$

Proof load: The maximum load which can be applied to a body without its permanent deformation is called proof load.

Strain Energy in Simple Tension and compression

Let us take the case of bar of cross-section area A and length l are subjected to load W. Suppose this load extend the bar by the amount δl and produced a maximum stress σ

The work done by W and hence strain energy U, stored in the material is equal to the area.

Strain energy stored in bar = Work done by the load

$$U = \frac{1}{2} W \,\delta l$$

$$U = \frac{W}{2} \frac{\sigma l}{E}$$

$$U = \frac{\sigma A}{2} \frac{\sigma l}{E}$$

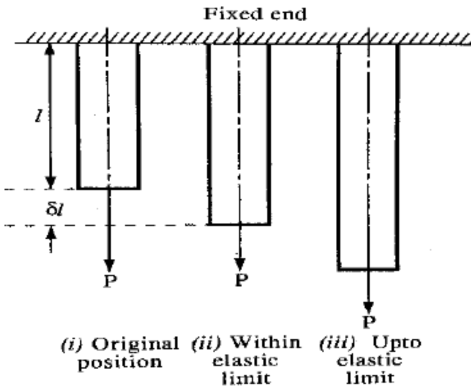
$$U = \frac{\sigma^2 A l}{2E}$$

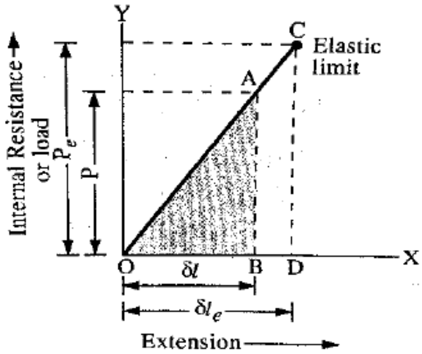
$$\boxed{U = \frac{\sigma^2 V}{2E}}$$

If σ_p be the proof Stress or maximum stress to which the bar is stress up to the elastic limit.

$$\text{Proof resilience } (U_p) = \frac{{\sigma_p}^2}{2E} \times V$$

$$\text{Modulus of resilience} = \frac{{\sigma_p}^2}{2E}$$





Stress Due to different type of Load:

1. Gradually Applied Loads.
2. Suddenly Applied Loads.
3. Falling or Impact Loads.

Stress Due to gradually applied load: A body is said to be acted upon by a gradually applied load if the load increase from zero and reaches its final value stepwise. Let W be the load applied gradually on a body and lets δl and be the corresponding change in length and maximum induced in it.

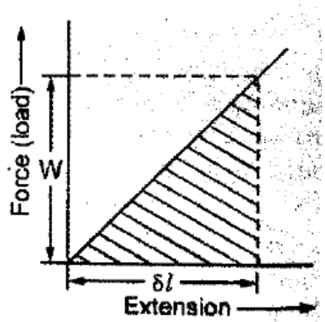
Energy due to external load = $\frac{1}{2}\ \sigma\ A\ \delta l$

Work done on the body = $\frac{1}{2}\ W\ \delta l$

Strain energy stored = work done on the body

$$\frac{1}{2}\ \sigma\ A\ \delta l = \frac{1}{2}\ W\ \delta l$$

$$\boxed{\sigma = \frac{W}{A}}$$



Stress Due to gradually applied loads: When the load is applied all the sudden and not stepwise, it is called suddenly applied load. Let W be the load applied suddenly on a body and lets δl and be the corresponding change in length.

Energy stored = external work done

$$W\ \delta l = \frac{1}{2}\ \sigma\ A\ \delta l$$

$$\boxed{\sigma = \frac{2W}{A}}$$

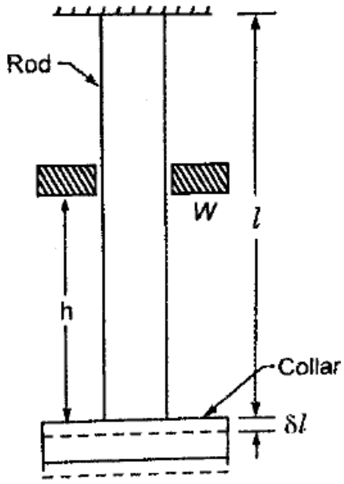
(Due to suddenly applied load is double that of gradually applied load.)

Stress Due to Impact loads: The load which are fall from a height or strike the body with certain momentum are called falling or impact loads.

Consider a weight W falling through a height h on a collar fitted on the rod which is of length l and has cross-section area A.

Let the extension and maximum stress produced be δl and σ_t respectively.

External work done on the bar = energy stored in the bar



$$W(h + \delta l) = \frac{1}{2}\ \sigma\ A\ \delta l$$

$$W\left(h + \frac{\sigma l}{E}\right) = \frac{1}{2}\ \sigma\ A \frac{\sigma l}{E}$$

$$W\left(h + \frac{\sigma l}{E}\right) = \frac{\sigma^2 Al}{2E}$$

$$\frac{\sigma^2 Al}{2E} - \frac{\sigma^2 Wl}{E} - Wh = 0$$

$$\sigma = \frac{\frac{Wl}{E} \pm \sqrt{\frac{W^2l^2}{E^2} \pm \frac{WhAl}{E}}}{\frac{Al}{E}}$$

$$\sigma = \frac{\frac{Wl}{E} \pm \frac{Wl}{E}\sqrt{1 \pm \frac{2WAl}{E} \times \frac{E^2}{W^2l^2}}}{\frac{Al}{E}}$$

$$\sigma = \frac{W + W\sqrt{1 \pm \frac{2hAE}{Wl}}}{A}$$

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 \pm \frac{2hAE}{Wl}}\right]$$

If δl is negligible as compared to h,

$$\sigma = \frac{2W}{A}$$

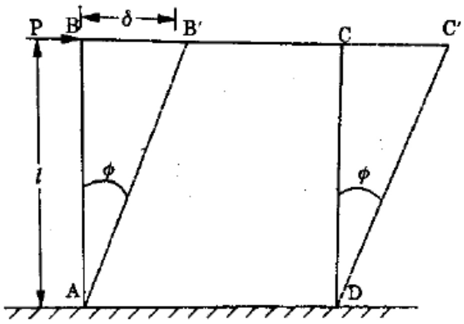
Strain Energy Stored in a body due shear stress:

Let us consider a cubical block ABCD of length l whose one face AD is fixed while shear force load P applied on the opposite face BC. Let τ is the shear stress induced and $\emptyset$ is the corresponding shear strain. Let G is the modulus of rigidity.

$$\text{Average Load} = \frac{0+P}{2}$$

Strain energy Stored U = work done in deforming the rectangular block

U = Average Load × Deformation



$$= \frac{P}{2} \times CC'$$

$$= \frac{P}{2} \times DC\,\emptyset$$

$$= \frac{1}{2}\tau \times (BC \times l) \times DC \times \emptyset$$

$$= \frac{1}{2}\tau \times \emptyset\,(BC \times DC \times l)$$

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$= \frac{1}{2}\tau \times \frac{\tau}{C} \times V$$

$$U = \frac{\tau^2}{2C} \times V$$

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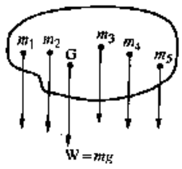
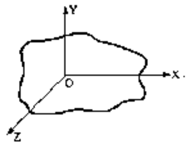
Chapter – 3

Moment of Inertia



Centre of gravity of a body(G): It is defined as the point of through which the whole weight of a body may be assumed to act. It denoted by C.G. and G. The position of C.G. depends upon the shape of the body.



Centroid: The centroid and centre of area is defined as the point where the whole area of the figure as rectangle, square, triangle, circle etc. is assumed to be concentrated. Thus centroid can be taken as quite analogous to centre of gravity when bodies have area only and not weight.

Moment of inertia (I): The moment of force about any point is the product of the force and the perpendicular distance between them. It also known as the first moment of force. If this first moment is again multiplied by the perpendicular distance between them the product so obtain is called second moment of force.

Moment if Inertia = Force × Square of perpendicular between them

$$I = Fy^2$$

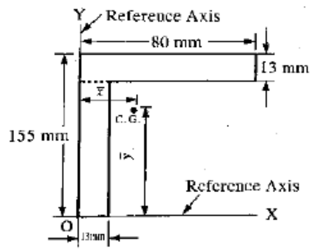
S.I. unit of moment of inertia is m^4.

Second moment of area: the product of the area (or mass) and the square of the perpendicular distance of the centre of gravity of the area (or mass) from reference axis is known as second moment of area.

S.I. unit Second of moment of area is m^4.

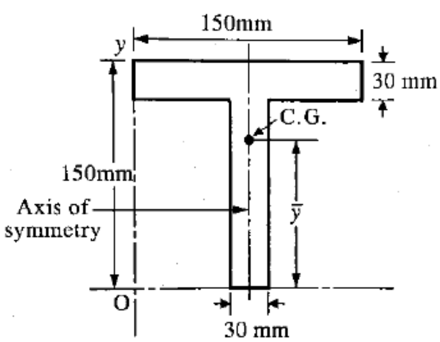
Moment if Inertia = Area of lamina × Square of perpendicular between them

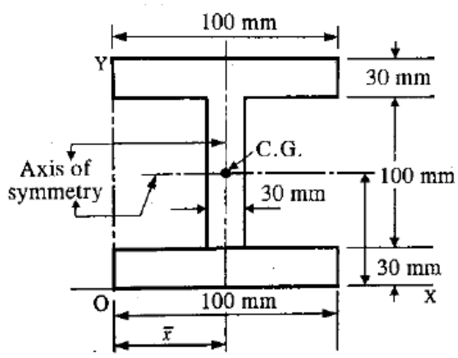
$$I = Ay^2$$



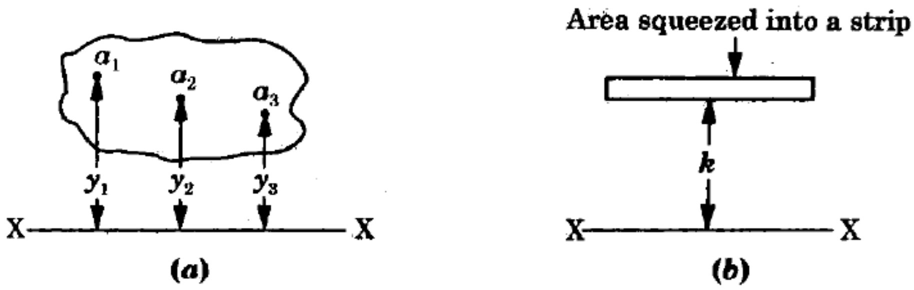
Axis of Reference: In case of the two dimensional figures the exact position of centre of gravity can be found by calculating the horizontal and vertical distances with reference to some assumed axis which are known as axis off reference.

Axis of symmetry: Axis of symmetry is that axis which divides the figure into halves which are same in size and shape. The C.G. of the figure lie on the axis of the symmetry.





Radius of gyration (K): it is defined as the distance from the axis of rotation at which, if the whole mass of the body is concentrated, its moment of inertia about the axis is the same as that which the actual distribution of mass.



Let us consider a body dividing in numbers of area a_1, a_2, a_3, and situated at the distance y_1, y_2, y_3 from the axis of rotation.

$$I_{xx} = \sum_{i=1}^{i=n} a_i y_i^2$$

Moment of inertia of area = $\sum ak^2$

$$I_{xx} = Ak^2$$

$$K = \sqrt{\frac{I_{xx}}{A}}$$

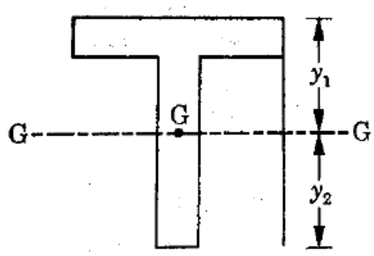
Where K is the radius of gyration.

- Minimum radius of gyration about X-axis (k_x) = $\sqrt{\frac{I_{xx}}{A}}$
- Minimum radius of gyration about Y-axis (k_y) = $\sqrt{\frac{I_{yy}}{A}}$

Section Modulus(Z): It is the ratio of moment of inertia of lamina about its centroidal axis and distance of the extreme fiber from centroidal axis is called section modulus. It is denoted by Z.

S.I. unit of section modulus is m^3.

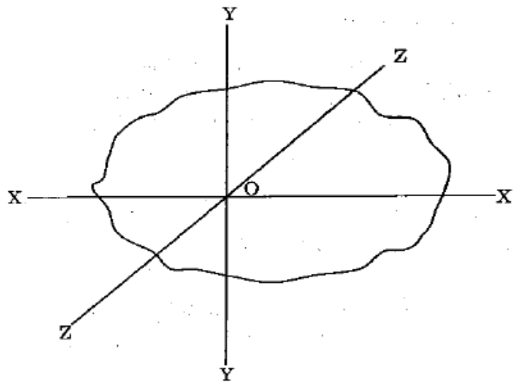
$$Z = \frac{I_{gg}}{y_2}$$



I_{gg} = Moment of inertia about centroidal axis GG.

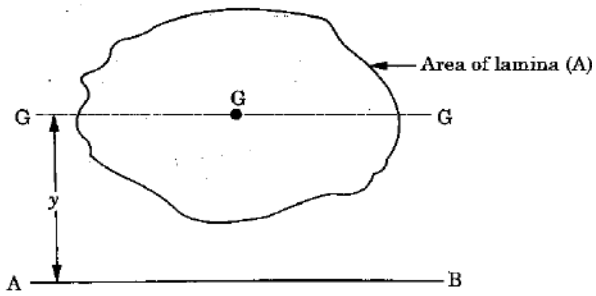
Theorem of perpendicular axis: If I_{xx} and I_{yy} be the moment of inertia about mutually perpendicular axes OX and OY in the plane of lamina and I_{zz} is the moment of inertia of the lamina about an axis OZ normal to the lamina and passing through the point of intersection of the axis OX and OY then

$$I_{zz} = I_{xx} + I_{yy}$$



Theorem of Parallel Axis: If the moment of inertia of a plane area about an axis in the plane of area through its C.G. be represented by I_G, then the moment of inertia of the given plane area about a parallel axis AB in the plane of area at a distance *y* from the C.G. of the area is given by

$$I_{AB} = I_G + Ay^2$$

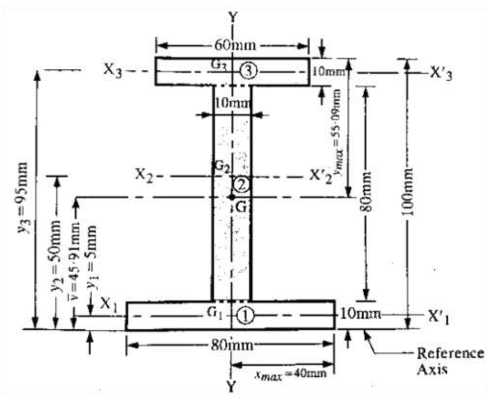


Formulas

Determine the C.G. about the X - axis and Y - axis

$$\text{C.G. about X- axis} = \overline{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$\text{C.G. about X- axis} = \overline{Y} = \frac{a^1y^1 + a^2y^2 + a^3y^3}{a^1 + a^2 + a^3}$$



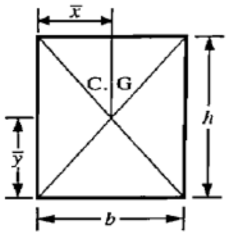
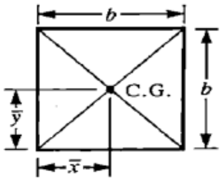
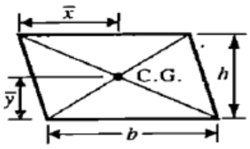
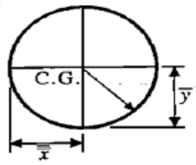
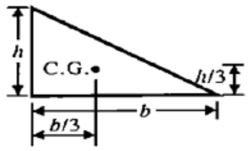
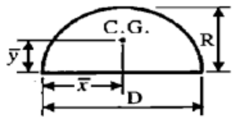
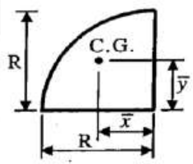
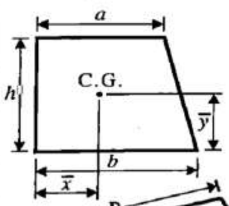
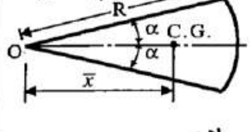
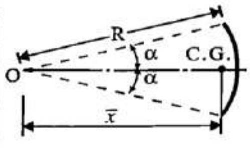
Determine the MOI about the I_{xx} and I_{yy}

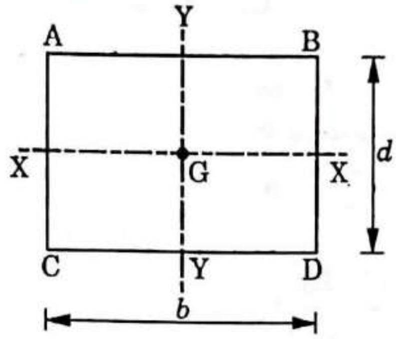
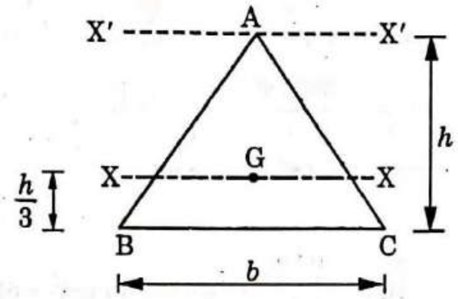
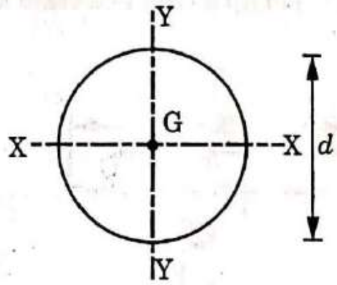
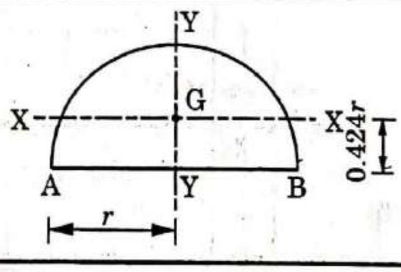
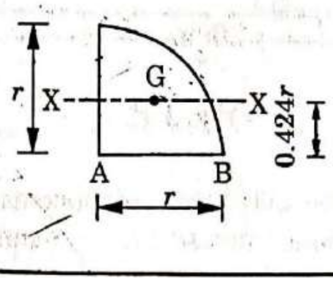
$$I_{xx1} = \frac{b_1{}^3 d_1{}^3}{12} + a_1(\overline{Y} - y_1)^2 \text{ (for single Rectangle part)}$$

$$I_{yy1} = \frac{d_1{}^3 b_1{}^3}{12} + a_1(\overline{X} - x_1)^2 \text{ (for single Rectangle part)}$$

Notes: Find MOI of individual rectangle figure and add this So get total MOI of given figure.

$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3} \qquad I_{yy} = I_{yy1} + I_{yy2} + I_{yy3}$$

S.No	Shape	Area	$\bar{x}$	$\bar{y}$	Figure	Position of C.G.
1.	Rectangle	$b \times h$	$b/2$	$h/2$		At intersection of diagonals
2.	Square	$b \times b$	$b/2$	$b/2$		At intersection of diagonals
3.	Parallelogram	$b \times h$	$b/2$	$h/2$		At point of intersection of its diagonals
4.	Circle	πR^2	R	R		At centre or at intersection of two diameters.
5.	Triangle	$\frac{1}{2} \times b \times h$	$\frac{b}{3}$	$\frac{h}{3}$		At intersection of medians
6.	Semi circle	$\frac{\pi R^2}{2}$	R	$\frac{4R}{3\pi}$ or 0·424R		$\frac{4r}{3\pi}$ on a line perpendicular to the diamet
7.	Quadrant	$\frac{\pi R^2}{4}$	0·424R	0·424R		At a point which is 0·424 R towards right and –0·424 R above base as shown.
8.	Trapezium	$(a+b)\frac{h}{2}$	$\frac{a^2+b^2+ab}{3ab}$	$\frac{(2a+b)}{a+b} \times \frac{h}{3}$		$\bar{x}$ and $\bar{y}$ as shown in Fig.
9.	Circular sector	αR^2	$\left\lvert\frac{2R\sin\alpha}{3\alpha}\right\rvert$	$\lvert 0 \rvert$		
10.	Arc of circle	$2\alpha R$,	$=\frac{R\sin\alpha}{\alpha}$	$\bar{y} = 0$,		

Name of the Figure	Shape	Moment of Inertia (I)
Rectangle		$I_{XX} = \frac{bd^3}{12}$ $I_{YY} = \frac{db^3}{12}$ $I_{CD} = \frac{bd^3}{3}$
Triangle		$I_{X'X'} = \frac{bh^3}{4}$ $I_{XX} = \frac{bh^3}{36}$ $I_{BC} = \frac{bh^3}{12}$
Circle		$I_{XX} = I_{YY} = \frac{\pi d^4}{64}$ $I_P = I_{ZZ} = \frac{\pi d^4}{32}$
Semi – circle		$I_{XX} = 0.11\ r^4$ $I_{AB} = \frac{\pi r^4}{8}$ $I_{YY} = \frac{\pi r^4}{8}$
Quadrant		$I_{XX} = 0.0549\ r^4$ $I_{AB} = \frac{\pi r^4}{16}$

E-Learning Notes

On

Strength of Material

BENDING STRESS

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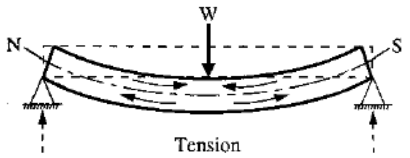
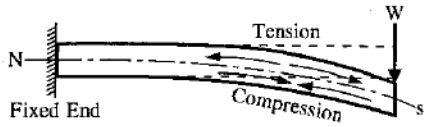
Chapter – 5

Bending Stress

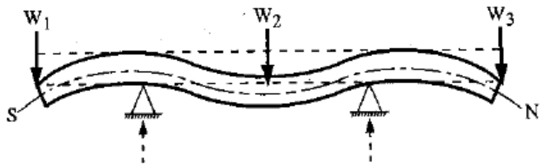
Bending Stress: The resistance induced to resist bending is known as bending stress or longitudinal stress.



Bending of simply supported beam



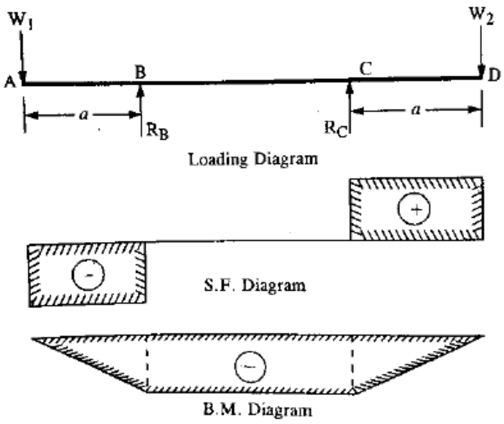
Bending of Cantilever

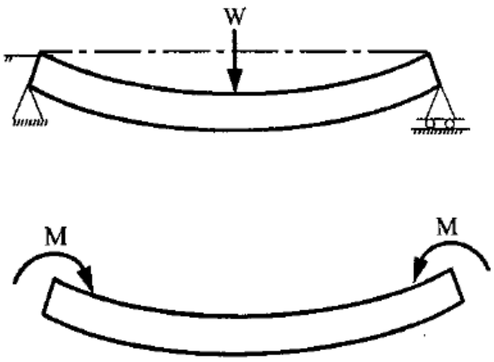


Bending of overhanging Beam

Shearing Stress: The resistance induced to resist shearing is known as shearing stress.

Pure bending: The bending of beam is not accompanied by any shear force is known as pure bending. Or simple bending.



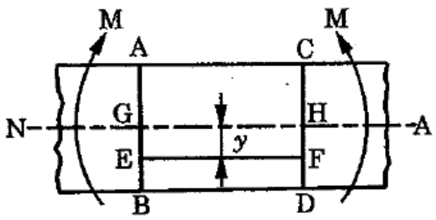


Assumption in the theory of simple banding:

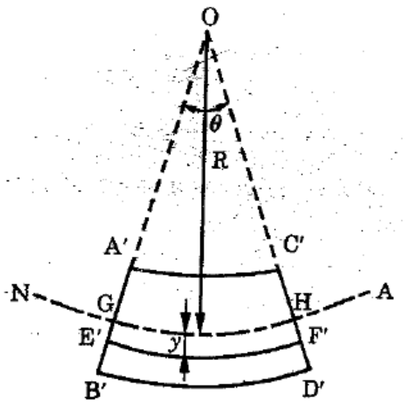
1) The plane transfer section remain plane even after bending.
2) The beam material is homogeneous and isotropic (*i.e.* same property everywhere in the material).
3) The modulus of elasticity has the same value for tension and compression.
4) The beam material stressed within the elastic limit and obeys hook's law.
5) There is no resultant push and pull on the beam section.
6) Each cross-section of the beam is symmetrical about the plane of bending.
7) The load are applied in the plane of bending.
8) The radius of curvature of the beam before bending is very large as compared to the transverse dimensions of the beams.

Bending Equation:

Let us consider a small portion ABCD of the beam which is subjected to bending moment M unaccompanied by any shear force between two parallel section AB and CD.



Beam before Bending



Beam after Bending

θ = angle by the arc at the centre

R = Radius of curvature

Let us consider a layer EF and at a distance y from the neutral axis. After deformation, EF will become E'F'.

$\therefore$ Change in layer of layer EF = E'F' – EF

= E'F' – GH (∵ EF = GH)

$= (R + y)\theta - R\theta$

$= R\theta + y\theta - R\theta$

$= y\theta$

Strain in layer EF, $\in = \dfrac{\text{Change in length of EF}}{\text{original length of EF}}$

$$= \frac{y\theta}{R\theta} = \frac{y}{R} \qquad \ldots(i)$$

But Strain, $\in = \dfrac{\sigma}{E}$ ...*(ii)*

From equation *(i) and (ii)*

$$\frac{y}{R} = \frac{\sigma}{E}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{E}{R} \times y$$

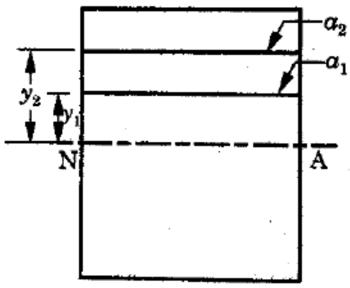
E and R are the constant, stress in any fibre is proportional to the distance of the fibre from the neutral axis.

Let us consider elementary areas a_1, a_2, …… etc. at a distance of y_1, y_2, …. etc. respectively from neutral axis. Let σ_1, σ_2…. be the stresses on elementary areas a_1, a_2, …… etc.

Force on elementary area $a_1 = \sigma_1 a_1$
Force on elementary area $a_2 = \sigma_2 a_2$

Moment of all these forces bout neutral axis

$$M = \sigma_1 a_1 y_1 + \sigma_2 a_2 y_2 + \ldots \qquad \ldots(iii)$$



$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{E}{R} \times y$$

$$\sigma_1 = \frac{E}{R} \times y_1$$

$$\sigma_2 = \frac{E}{R} \times y_2$$

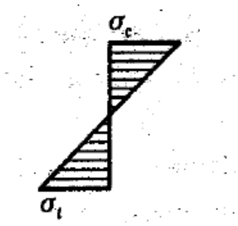
Putting the value of σ_1, σ_2 in equation *(iii)*

$$M = \frac{E}{R} y_1 a_1 y_1 + \frac{E}{R} y_2 a_2 y_2 + \ldots\ldots\ldots$$

$$M = \frac{E}{R} a_1 y_1^2 + \frac{E}{R} a_2 y_2^2 + \ldots\ldots\ldots$$

$$M = \frac{E}{R} \left(a_1 y_1^2 + a_2 y_2^2 + \ldots\ldots\ldots\right)$$

$$M = \frac{E}{R} I$$



Stress Diagram

I = moment of inertia of beam section about neutral axis

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

This is the bending equation

Moment of resistance: The sum of moments of the internal forces about neutral axis is known as the moment of resistance of flexural strength.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma}{y} I = \sigma \times \frac{I}{y}$$

$$\boxed{M = \sigma Z}$$

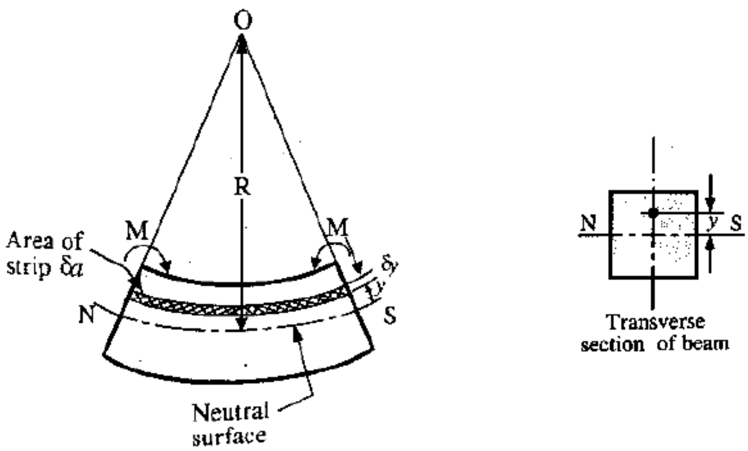
Z is known as section modulus.

Section Modulus: It is ratio of moment of inertia od section about the neutral axis to the distance from the extreme fibre from the neutral axis.

$$Z = \frac{I}{y}$$

I = moment of inertia of the beam
y = distance of the extreme fibre from the neutral axis

Neutral axis: the intersection of neutral surface with the cross- section of beam perpendicular to its longitudinal axis is called neutral axis. Neutral axis of a beam is the axis at which is stress is zero.



Modulus of Rupture

If M is the bending moment at which the beam fails due to bending, then the value of calculated by formula is called modulus of rupture of the material.

Formula

$$\sigma = \frac{M}{Z}$$

Flexural Formula

Flexural formula can be used to compute the maximum stress due to bending.

$$\sigma_{max} = \frac{My}{I}$$

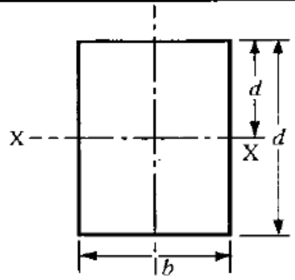
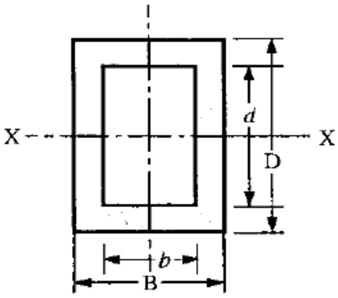
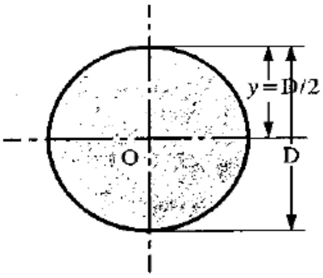
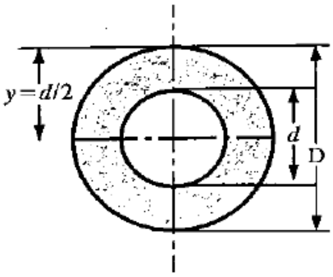
σ_{max} = Maximum stress at the outer most fibre of the beam

M = Bending moment

ending moment

y = distance from the neutral axis of the beam

I = Moment of inertia of the cross- section

Section		Section Modulus
Rectangular Solid section		$I_{XX} = \frac{bd^3}{12}$, $y_{max} = d/2$ $Z_{XX} = \frac{I_{XX}}{y_{max}} = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}$
Hollow Rectangular Section		$I_{XX} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{BD^3 - bd^3}{12}$ $y_{max} = \frac{D}{2}$ $Z_{XX} = \frac{I_{XX}}{y_{max}} = \frac{(BD^3 - bd^3)/12}{D/2}$ $Z_{XX} = \frac{BD^3 - bd^3}{6D}$
Circular Solid Section		$I = \frac{\pi}{64} D^4$, $y_{max} = D/2$ $Z = \frac{I}{y_{max}} = \frac{\pi/64 D^4}{D/2} = \frac{\pi D^3}{32}$
Hollow Circular section		$I = \frac{\pi}{64}(D^4 - d^4)$, $y_{max} = \frac{D}{2}$ $Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}(D^4 - d^4)}{D/2}$ $Z = \frac{\pi(D^4 - d^4)}{32D}$

Formulas

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$Z = \frac{I}{y}$$

$$\sigma_{max} = \frac{My}{I}$$

$$\sigma = \frac{M}{Z}$$

$$M = \sigma Z$$

$$I_{xx} = \frac{bd^3}{12}$$

$$I_p = \frac{\pi D^4}{32} \text{ For Solid Shaft}$$

$$I_p = \frac{\pi}{32}(D^4 - d^4) \text{ For Solid Hollow Shaft}$$

$$I_{yy} = \frac{db^3}{12}$$

$$M = \frac{Wl}{4} \quad \text{(for point load condition)}$$

$$M = \frac{Wl^2}{8} \quad \text{(for UDL condition)}$$

E-Learning Notes

On

Strength of Material

COLUMN AND STRUT

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Chapter – 6

Column and Strut

Strut: The structural member subjected to an axial compressive load in any position other than the vertical and fixed rigidly or hinged and pin jointed at one or both the ends is called strut. Ex. Connecting rod.

Column: The structural members subjected to an axial compressive load inclined at 90° to the horizontal position is called column. Ex. Vertical pillar between roof and floor.

Slenderness Ratio: The ratio of length of column to the minimum radius of gyration of cross-section area of the column in known as Slenderness Ratio.

$$\text{Slenderness ratio} = \frac{\text{length of column}}{\text{Minimum radius of gyration}} = \frac{l}{K_{min}}$$

$$K_{min} = \text{minimum ratio of gyration} = \sqrt{\frac{I_{min}}{A}}$$

I_{min} is selected lower value from I_{xx} and I_{yy}

Buckling Load: The minimum limiting load at which the column tends to have lateral displacement or tends to buckle is called a buckling or crippling or critical load.

Safe Load: The load to which a column is subjected and which is below the buckling load is called safe load.

$$\text{Safe load} = \frac{\text{Buckling Load}}{\text{Factor of safety}}$$

Equivalent length: Equivalent or effective length of the column is the length which give the same buckling load as given by a both ends hinged column.

Crushing Load: The maximum axial compressive load which a column can take without failure by crushing is called crushing load. Short column fails due to crushing load.

Classification of column

Column are classified in to three classes

1) Short column
2) Medium or intermediate column
3) Long column

1) Short column: The which have length is less than 8 times of their diameter or slenderness ratio less than 32 are called short column.
2) Intermediate column: The column which have length varying from 8 to 30 times of their diameter or their slenderness ratio varying from 32 to 120 are known as medium or intermediate column.

3) Long column: The column which have length is more than 30 times of their diameter or slenderness ratio more than 120 is called Long column.

Strength of column: The strength of column depends upon the following factors:

1. End condition
2. Slenderness ratio of the column
3. Material of the column

As the slenderness ratio increases, the compressive strength of column decrease and tendency of buckling increase.

End Condition of column

1. Both ends are hinged or pin joined
2. Both ends fixed
3. One ends fixed and other hinged
4. One ends fixed and other free

1. Both ends of column are hinged

The ends of column do not move or have their lateral displacement. The column is free to slope when it buckles.

Equivalent length = Actual length

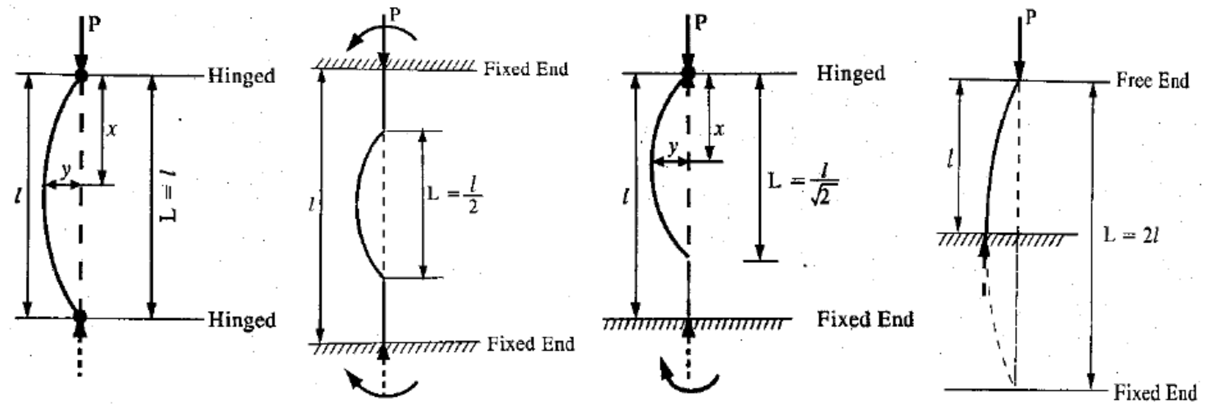
$$L = l$$

2. Both ends fixed

The both ends of column are rigidly fixed *i.e.* these ends neither move nor take any slope.

$$\text{Equivalent length} = \frac{\text{Actual length}}{2}$$

$$L = \frac{l}{2}$$



Both ends hinged Joint | **Both ends fixed** | **One ends fixed and other hinged** | **One ends fixed and other free**

3. **One ends fixed and other hinged**

The one end of the column is fixed *i.e.* it neither move nor does it takes any slope. The other ends are hinged *i.e.* it does not move but free to slope.

$$\text{Equivalent length} = \frac{\text{Actual length}}{\sqrt{2}}$$
$$L = \frac{l}{\sqrt{2}}$$

4. **One ends fixed and other free**

One ends is fixed rigidly *i.e.* it does not move and does not take any slope. The other ends is free to take up any position and slope.

$$\text{Equivalent length} = 2 \times \text{Actual length}$$
$$L = 2 \times l$$

Equivalent length of column

The equivalent length of given column with the ends condition in the length of an equivalent column of the same material and cross- section with hinged ends having the value of buckling load is equal to that of the given column.

l = Actual length of column
L = equivalent length of column

1. When the both ends hinged	$L = l$
2. When both ends fixed	$L = l/2$
3. When one ends is fixed and other hinged	$L = l/\sqrt{2}$
4. When one ends is fixed and other is free	$L = 2l$

Euler's theory of long column

Assumptions:

1) The material of column s homogeneous and isotropic *i.e.* the elastic properties are same in all the direction.
2) The length off column is more than 30 times of its diameter.
3) The cross- section of column is uniform throughout its length.
4) The self-weight of column is neglected.
5) The column failed by buckle alone.
6) The direct stress is very small as compare to the bending stress.

The **Euler's formula** is given by

$$P_e = \frac{\pi^2 EI}{L^2}$$

P_e = Buckling load or critical load
E = modulus of elasticity
I = minimum value of MOI (I_{xx} and I_{yy})
L = equivalent length of column

Sr. no.	End condition of column	Relation between effective length (L) And Actual Length (l)	Euler's Buckling load in tem of actual length
1	Both ends hinged	$L = l$	$\frac{\pi^2 EI}{l^2}$
2	Both ends fixed	$L = \frac{l}{2}$	$\frac{4\pi^2 EI}{l^2}$
3	One ends is fixed and other hinged	$L = \frac{l}{\sqrt{2}}$	$\frac{2\pi^2 EI}{l^2}$
4	One ends is fixed and other is free	$L = 2l$	$\frac{\pi^2 EI}{4l^2}$

Rankine Gordon formula

Euler's formula is valid for only long column. Rankine proposed an empirical formula for column which cover all type of column from very short to long column.

Very short column is subjected to axial compressive load fail by crushing load

$P_c = \sigma_c A$ (∴ σ_c is ultimate compressive strength)

Long column or strut fail by buckling

$$P_e = \frac{\pi^2 EI}{L^2}$$

The buckling load or crippling load by Rankine formula

$$\frac{1}{P_r} = \frac{1}{Pc} + \frac{1}{Pe}$$

P = Crippling load by Rankine formula
P_c = Crushing or compressive load
P_e =Crippling load obtain by Euler's formula

By Rankine Gordon formula

$$\frac{1}{P_R} = \frac{P_e + P_c}{P_c P_e}$$

$$P_R = \frac{P_e P_c}{P_c + P_e}$$

$$P_R = \frac{P_c}{1 + \frac{P_c}{P_e}}$$

$$P_R = \frac{\sigma_c A}{1 + \frac{\sigma_c AL^2}{\pi^2 EI}}$$

Putting $I = Ak^2$ where k is the least radius of gyration of the column section.

$$P_R = \frac{\sigma_c A}{1 + \frac{\sigma_c A}{\pi^2 E}\left(\frac{L}{k}\right)^2}$$

$$\boxed{P_R = \frac{\sigma_c A}{1 + \alpha\left(\frac{L}{k}\right)^2}}$$

where $\alpha = \frac{\sigma_c}{\pi^2 E}$ is a constant called Rankine constant.

Rankine Constant

Material	σ_c (N/mm^2)	Constant α (for both ends hinged)
Cast Iron	550	$\frac{1}{1600}$
Wrought Iron	250	$\frac{1}{9000}$
Aluminum	119.1	$\frac{1}{5000}$
Mild steel	320	$\frac{1}{7500}$
Timber	473.6	$\frac{1}{2000}$
Medium carbon steel	500	$\frac{1}{5000}$

Rankine formula for long column: In case of long column, the value of L is large, therefore value of P_e will be small. Hence the value of $\frac{1}{Pe}$ will be large enough as compared with $\frac{1}{Pc}$. Hence the value $\frac{1}{Pc}$ of may be neglected in Rankine formula.

$$\frac{1}{P_R} = \frac{1}{Pe}$$

$$P_R = P_e$$

Rankine formula for short column: In case of short column, the value of L is small, therefore, value of P_e will be large. Hence, value of $\frac{1}{Pe}$ will be small enough as compared to the value of $\frac{1}{Pc}$. Hence value of $\frac{1}{Pe}$ may be neglected in the Rankine formula.

$$\frac{1}{P_R} = \frac{1}{Pc}$$

$$P_R = P_C$$

E-Learning Notes

On

Strength of Material

TORSION

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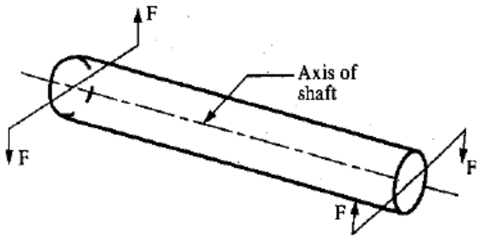
Chapter – 7

Torsion

Torque or Twisting Moment: The product of tangential force and the distance between the point of application of the force and the axis of the shaft is called twisting moment or turning moment and simply torque. It is denoted by T.

$$T = F \times r$$



Due to the torque the shear stresses are induced in the cross-section of the shaft. Such stresses are called "Torsional Shear stress".

Unit of torque: S.I. Unit of torque is N-m.

Torsion: The twisting moment produced in the shaft when a force acts tangentially to the circumference of the shaft is known as torsion. The torsion measured in degrees and radians.

Angle of twist: the angle through which the cross- section of the shaft is rotated by the action of twisting moment or torque is called angle of twist.

Pure torsion: A shaft is said to be pure torsion if it is subjected to equal and opposite end torques whose axes coincide with the axis of the shaft.

Young modulus or Modulus of elasticity: It is ratio of tensile stress to the tensile strain It is denoted by (E).

$$\text{Young Modulus (E)} = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\sigma_t}{\epsilon_t}$$

Shear modulus or Modulus of rigidity: It is ratio of Shear stress to the Shear strain It is denoted by (G).

$$\text{Shear Modulus (G)} = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

Difference Between Torque and Torsion

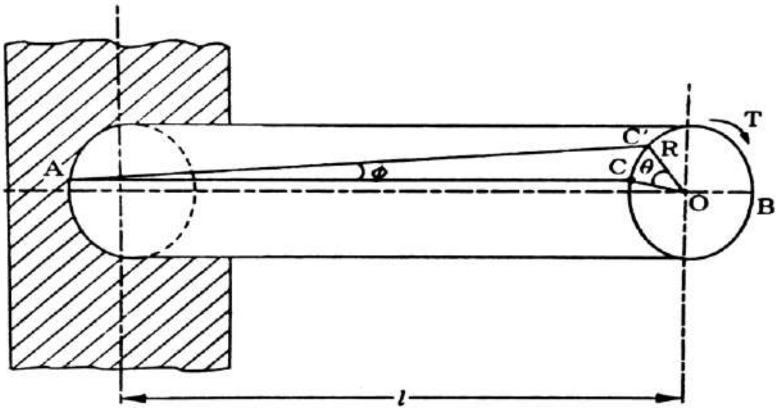
Sr. no.	Torque	Torsion
1.	It is the effect of force about any point *i.e.* Applying force at some distance from the given point.	It is the phenomenon is applying torque on shaft *i.e.* applying equal and opposite torque on both end of the shaft.
2.	It is measured in N-m.	It has no physical unit but measured in degrees and radians.
3.	It is related with many physical works like opening of door and wrench it.	It is related with only shaft.
4.	Torque can be measured.	Torsion cannot be measured.
5.	It inly occurs when force is applied.	It only occurs where torque is applied.
6.	Only one force is needed at a distance from the given point.	Torque is needed at both ends of the shaft.

Assumptions in the theory of pure torsion

1. The material of shaft is homogeneous and isotopic *i.e.* having same elastic properties in all the directions.
2. The cross- section of the shaft in uniform throughout.
3. Maximum shear stress induced in the shaft does not exceed its elastic limit value.
4. The twist along the length of the shaft in uniform throughout.
5. The shaft is straight and circular in section throughout and its length remain circular after loading.
6. The distance between any two normal cross-section remain the same after the application of torsion.
7. The shaft is under pure torsion *i.e.* the pure twisting moment is applied to the shaft and it acts in plane perpendicular to the axis of the shaft.

Torsion equation for a solid shaft

T = Torque or twisting moment
R = radius of shaft.
l = length of the shaft.
G = modulus of rigidity.
τ = maximum shear stress at the surface of the shaft.
θ = angle of twist.
I_p =Polar moment of inertia of the shaft.



The shear strain is deformation per unit length.

$$\text{Shear strain}(\emptyset) = \frac{CC'}{AC}$$

$$= \frac{CC'}{l} = \tan\emptyset = \emptyset \qquad (\text{ is very small angle} \therefore \tan\emptyset = \emptyset)$$

But arc CC' = $R\theta$

$$\text{Shear strain}(\emptyset) = \frac{R\theta}{l} \qquad ...(i)$$

Also

$$\text{Shear strain}(\emptyset) = \frac{\text{Shear Stress}}{\text{Modulus of rigidity}}$$

$$\emptyset = \frac{\tau}{G} \qquad ...(ii)$$

From equation *(i)* and *(ii)*

$$\frac{\tau}{G} = \frac{R\theta}{l}$$

$$\frac{\tau}{R} = \frac{G\theta}{l} \quad \text{...(iii)}$$

$$\tau = \frac{G \times \theta \times R}{l}$$

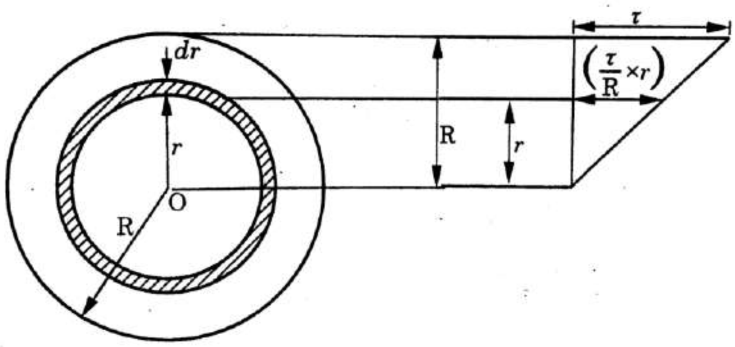
$$\tau = \frac{G\theta}{l} \times R$$

$$\tau = \text{Constant} \times R$$

$$\tau \propto R$$

$$\frac{\tau}{R} = \text{Constant}$$

The shear stress at any section in the shaft is directly proportional to the distance of the section from the axis of the shaft. Maximum shear stress will be at the outer surface and minimum shear stress (*i.e.* zero) will be at the axis of the shaft.



Let us consider an elementary circular ring of thickness *dr* at a distance *r* from the centre of the shaft.

Shear stress at a distance *r* from the centre of the shaft = $\frac{\tau}{R} \times r$

Area of elementary circular ring = $2\pi r\, dr$

∴ Shear resistance of the elementary ring = shear stress × area of elementary ring

$$= \left(\frac{\tau}{R} \times r\right) \times (2\pi r\, dr)$$

$$= \frac{\tau}{R} \times 2\pi r^2\, dr$$

Turning moment of shear resistance offered by elementary ring

$$dT = \frac{\tau}{R} \times 2\pi r^2\, dr \times r$$

$$= \frac{\tau}{R} \times 2\pi r^3\, dr$$

Total turning moment = summation of the turning moment of shear resistance offered by all the elementary rings.

$$T = \int_0^R dT$$

$$T = \int_0^R \frac{\tau}{R} \times 2\pi r^2\, dr$$

$$T = \frac{\tau}{R} \times 2\pi \left|\frac{r^4}{4}\right|_0^R$$

$$T = \frac{\tau}{R} \times 2\pi \times \frac{R^4}{4}$$

$$T = \frac{\tau}{R} \times 2\pi \frac{1}{4}\left(\frac{D}{2}\right)^4 \qquad \left(\because R = \frac{D}{2}\right)$$

$$T = \frac{\tau}{R} \times 2\pi \frac{D^4}{4\times16}$$

$$T = \frac{\tau}{R} \times \frac{\pi D^4}{32} \qquad \left(\because \frac{\pi D^4}{32} = Ip\right)$$

$$T = \frac{\tau}{R} \times I_p$$

$$\frac{T}{Ip} = \frac{\tau}{R} \qquad ...(iv)$$

From equation *(iii)* and *(iv)*

Torsion Equation → $$\boxed{\frac{T}{Ip} = \frac{\tau}{R} = \frac{G\theta}{l}}$$

Strength of Solid shaft

The maximum torque or power transmission by a solid shaft is known as strength of solid shaft.

T = maximum torque transmitted by the shaft

D= diameter of the shaft

τ = maximum shear stress at the outer surface of the shaft

$$\frac{T}{Ip} = \frac{\tau}{R}$$

$$T = \frac{\tau}{R} \times I_p$$

$$T = \frac{\tau}{D} \times 2 \times \frac{\pi D^4}{32}$$

$$\boxed{T = \frac{\pi}{16}\tau D^3}$$

Strength of Hollow shaft

The maximum torque or power transmission by a hollow shaft is known as strength of hollow shaft.

T = maximum torque transmitted by the shaft

D= Outer diameter of the shaft

d= Inner diameter of the shaft

τ = maximum shear stress at the outer surface of the shaft

$$\frac{T}{Ip} = \frac{\tau}{R}$$

$$T = \frac{\tau}{R} \times I_p$$

$$T = \frac{\tau}{D} \times 2 \times \frac{\pi}{32}(D^4 - d^4)$$

$$\boxed{T = \frac{\pi}{16}\tau\left(\frac{D^4 - d^4}{D}\right)}$$

Torsional rigidity of a shaft

The torque required to produce a twist of one radian per unit length of the shaft.

$$\frac{T}{I_p} = \frac{G\theta}{l}$$

$$T = \frac{G\theta}{l} \times I_p$$

θ = 1 unit and l = 1 unit then T = Torsional of rigidity

Torsional of rigidity, $T = GI_p$

Power transmission by the shaft

The main purpose of a shaft is to transmit power from one shaft to another.

N = number of revolution of the shaft per minutes.

T = average or mean torque transmitted by the shaft.

∴ Work done by the shaft per second

= Mean torque × average velocity of the shaft

$= T \times \frac{2\pi N}{60}$

$= \frac{2\pi NT}{60}$ watts

Power transmission by the shaft, $P = \frac{2\pi NT}{60}$ watts

1 H.P. = 735.5 watts

Formulas

$$\frac{T}{Ip} = \frac{\tau}{R} = \frac{G\theta}{l}$$

$$T = F \times r$$

1 Pa(Pascal) = 1 N/m^2

1 MPA = 1 N/mm^2

1 GPA = $10^3 \times N/mm^2$

= 1 KN/mm^2

$$T = \frac{\pi}{16}\tau D^3$$

$$T = \frac{\pi}{16}\tau\left(\frac{D^4 - d^4}{D}\right)$$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{G\theta}{l} \times I_p$$

$$I_p = \frac{\pi D^4}{32}$$

$$I_p = \frac{\pi}{32}(D^4 - d^4)$$

E-Learning Notes

On

Strength of Material

SPRING

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Chapter – 8

Spring

Spring

- A **spring** is an elastic member which can be absorb energy by very last change in its form and then release then the same required.
- **Spring** are elastic member which deform and deflect under load by storing energy in them and regain their original shape when the load is removed.

Function of spring

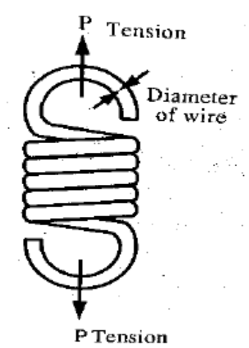
1) They are used to measure the forces as in spring balances.
2) They are used to store energy as in clock spring.
3) They are used absorb shock or impact loading as in carriage spring.
4) They are used to apply forces and to control motions as in brake and clutches.

Types of spring

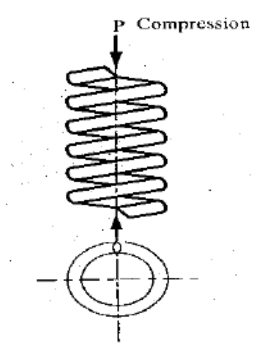
1) Helical spring
   a) Close coil Helical spring
   b) Open coil Helical spring
2) Laminated spring (leaf spring)
   a) Full elliptical spring
   b) Semi elliptical spring
   c) Cantilever
3) Spiral spring

Helical Spring: A length of wire when wound in helix forms a helical spring.

1) **Closed coil helical spring**: Closed coil helical spring are coiled in such a way that the coils are in contact with each other and slope of helix is so small that bending effect can be neglected. Closed coil are used mostly in tensile load only, because coils are contact with each other.



Tension Helical Spring
(Closed Coil Helical Spring)



Compression Helical spring
(Open Coil Helical Spring)

2) **Open coil helical spring:** Open coil helical spring there is a large gap between two consecutive coil. These spring are used in compressive as well as tensile load. The are subjected in bending load.

Terms related to helical spring

1) **Stiffness of a spring (S):** It is defined as the load per unit deflection. It is denoted by S. It is also known as spring constant.

$$S = \frac{W}{\delta}$$

W = Axial load on spring

δ = Deflection of spring due to axial load W

2) **Helix angle**: It is the angle which the axis of the wire makes with a horizontal line perpendicular to the axis of the spring.
3) **Solid length of a spring:** The solid length of spring means the distance between the coils when the coils are touching each other *i.e.* there is no gap between the spring coils.

Solid Length = Number of coils × Diameter of wire

4) **Spring Index:** It is ratio of the means coil diameter (D) to the diameter of spring wire (d).

$$\text{Spring Index} = \frac{D}{d}$$

5) **Angle of twist ($\boldsymbol{\theta}$):** it is the angle through which cross-section of a bar is twisted due to twisting moment or torque.
6) **Resilience (U):** When a body stresses with in the elastic limit, the amount of energy stored is called resilience or strain energy.
7) **Proof load:** It is maximum load to which the spring can be subjected without undergoing permanent deformation.
8) **Proof stress:** It is maximum stress to which the spring can be subjected without undergoing permanent deformation.
9) **Proof resilience:** It is maximum resilience of the spring without the occurrence of permanent deformation in the spring.
10) **Free length:** The length of spring in the free or unloaded condition is called free length.

Closed Coil Helical Spring

Let us consider a helical spring is subjected to axial load W. The upper end of the spring is fixed due to axial load.

- Twisting moment (T=WR) will act on spring and generated shear stress.
- Due to shear load W, bending stress will be induced in the wire of spring. Bending stress is very small as compared to shear stress. Hence bending stress is neglected.

d = Diameter of spring

n = Number of coil

D = Mean diameter of spring coil

R = Mean radius of spring coil $= \frac{D}{2}$

θ = Angle of twist in spring

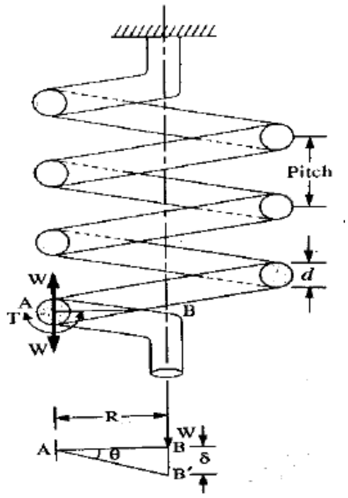
δ = Deflection of spring

l = Total length of spring wire

τ = Maximum shear stress induced in spring wire = $2\pi R_n$

The torsion equation

$$\frac{T}{Ip} = \frac{\tau}{r} = \frac{G\theta}{l}$$



Twisting moment on the wire, T =WR

Polar Moment of inertia of wire, $I_p = \frac{\pi d^4}{32}$

From torsion equation $\frac{T}{Ip} = \frac{\tau}{r}$

$$\frac{WR}{\frac{\pi d^4}{32}} = \frac{\tau}{d/2}$$

$$\tau = \frac{16WR}{\pi d^3} \quad \text{...(i)}$$

Further, $\frac{T}{Ip} = \frac{G\theta}{l}$

$$\frac{WR}{\frac{\pi d^4}{32}} = \frac{G\theta}{2\pi Rn}$$

$$\theta = \frac{64WR^2n}{Gd^4} \quad \text{...(ii)}$$

Deflection of spring, $\delta = R \times \theta$

$$\delta = R \times \frac{64WR^2n}{Gd^4}$$

$$\delta = \frac{64WR^3n}{Gd^4} \quad \text{...(iii)}$$

Energy stored in the spring, $U = \frac{1}{2} \times T \times \theta$

$$U = \frac{1}{2} \times WR \times \frac{64WR^2n}{Gd^4}$$

$$U = \frac{32W^2R^3n}{Gd^4} \quad \text{...(iv)}$$

Futher, $U = \frac{32W^2R^3n}{Gd^4}$

$$U = \frac{32R^3n}{Gd^4} \times \left(\frac{\tau \times \pi d^3}{16R}\right)^2$$

$$U = \frac{\tau^2}{8G} \times \pi^2 d^2 R_n$$

$$U = \frac{\tau^2}{4G}\left(2\pi Rn \times \frac{\pi d^2}{4}\right)$$

$$U = \frac{\tau^2}{4G} \times \text{Volume of spring wire}$$

$$U = \frac{\tau^2}{4G} \times V \qquad ...(v)$$

Stiffness of the spring, $S = \frac{W}{\delta} = \frac{W}{64WR^3n} \times Gd^4$

$$S = \frac{Gd^4}{64R^3n} \qquad ...(vi)$$

Let us consider a closed coil helical spring deflected due to impact loading falling from a height.

Let W be the weight falling from height h and W_e is the equivalent steady load producing same effect.

$$\frac{1}{2} W_e \times \delta = W\,(h + \delta)$$

$$\delta = \frac{64W_eR^3n}{Gd^4}$$

Spring in series:

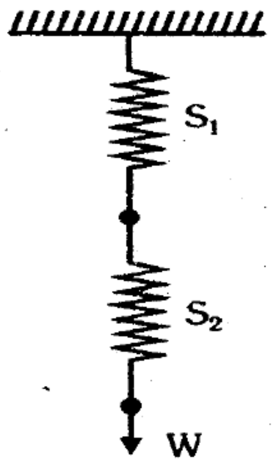
When two or more spring are joined end to end, they are said to be in series.

Total Extension, $\delta = \delta_1 + \delta_2$

$$\frac{W}{S} = \frac{W}{S_1} + \frac{W}{S_2}$$

$$\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}$$

i.e. reciprocal of stiffness of combination is the sum of reciprocals of individual stiffness.



Spring in series

Spring in parallel:

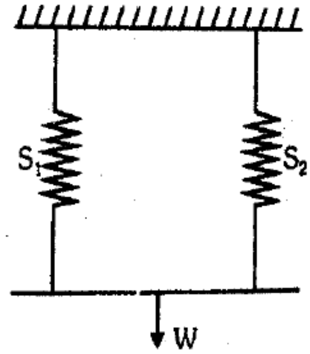
When two or more spring are connected in such a way that they share the load and deflect equally, then they are said to be parallel.

Total Load, $W = W_1 + W_2$

$\delta S = \delta S_1 + \delta S_2$

$S = S_1 + S_2$

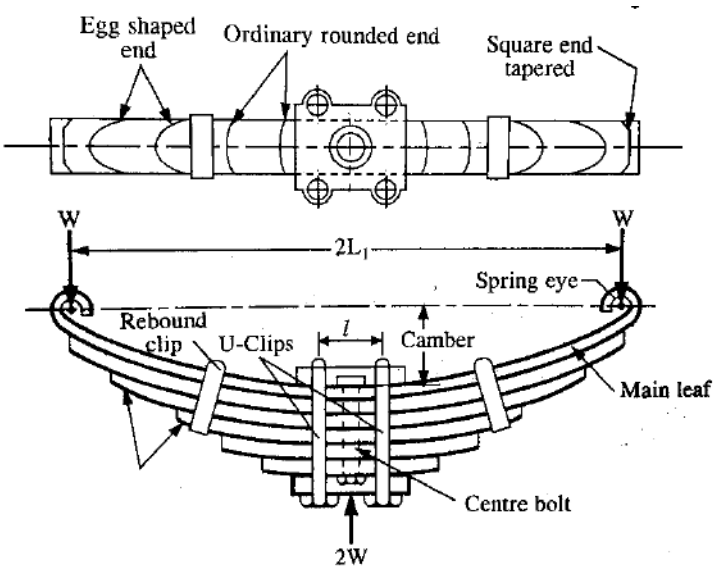
i.e. stiffness of combination is the sum of individual stiffness.



Spring in parallel

Leaf Spring or Laminated Spring:

Leaf Spring or laminated spring consist of a number of thin plates of same width and thickness, but of different lengths, all bent to the same curvature. Leaf spring are widely used in railways carriages, motor vehicle and trollies etc.

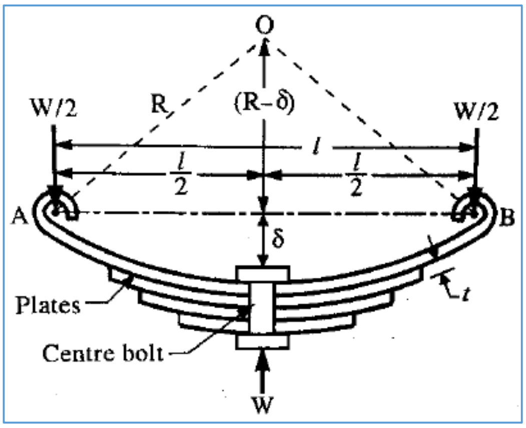


W = maximum load on the spring
l = length of spring
t = Thickness of leaves or plates
b = width of leaves or plates
n = Number of leaves or plates
R = Radius of curvature of plates
δ = Deflection of top laminate
σ = Maximum bending stress induced in the plates

Maximum bending moment at the centre, $M = \frac{Wl}{4}$

Section Modulus of each plate, $Z = \frac{I}{y} = \frac{\frac{1}{12}bt^3}{\frac{t}{2}} = \frac{bt^2}{6}$

Section modulus od n plates, $Z = \frac{nbt^2}{6}$



Laminated spring

We know that

$$M = \sigma Z$$

$$\frac{Wl}{4} = \sigma \times \frac{nbt^2}{6}$$

$$\sigma = \frac{3}{2} \times \frac{Wl}{nbt^2} \qquad \text{...(vii)}$$

Initial deflection $$\delta = \frac{l^2}{8R} \qquad \text{...(viii)}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{\sigma}{\frac{t}{2}} = \frac{E}{R}$$

$$R = \frac{Et}{2\sigma}$$

From equation $$\delta = \frac{l^2}{8 \times \frac{Et}{2\sigma}} = \frac{\sigma l^2}{4Et} \qquad \text{...(ix)}$$

Substituting the value of σ from equation *(vii)* to equation *(ix)*

$$\delta = \frac{3}{2} \times \frac{Wl}{nbt^2} \times \frac{l^2}{4Et} = \frac{3}{8} \times \frac{Wl^3}{nEbt^3}$$

$$W = \frac{8}{3} \times \frac{nEbt^3\delta}{l^3}$$

Stiffness of spring, $$S = \frac{W}{\delta} = \frac{8}{3} \times \frac{nEbt^3}{l^3} \qquad \text{...(x)}$$

Strain energy stored, $$U = \frac{1}{2} \times W\delta$$

$$U = \frac{1}{2} W \left(\frac{3}{8} \times \frac{Wl^3}{nEbt^3}\right)$$

$$U = \frac{3W^2l^3}{16nEbt^3}$$

$$U = \left(\frac{9}{4}\frac{W^2l^3}{n^2b^2t^4}\right) \times \frac{1}{6E} \times \left(\frac{n}{2}\ lbt\right)$$

$$U = \frac{\sigma^2}{6E} \times \text{Volume of spring}$$